RANDOM ELEMENTS IN LINEAR SPACES

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1. Introduction

For different applications it is necessary to consider random elements which take values in linear spaces which are not Banach spaces. On the other hand, because from a physical point of view these elements are observed with the help of some "instruments," two spaces have to be considered; the space E in which the random element takes its values, and the space F in which the instruments are defined. The case of linear instruments is particularly important. A mathematical theory of such a situation was proposed by Gelfand, Itô, Minlos, and others. During the past few years this theory was generalized by S. Ahmad [1] and A. Badrikian [2].

All the spaces considered in this paper are real linear spaces and all the topologies are separated, locally convex topologies. By measure, we shall always mean probability measure (that is, positive measure with total mass equal to one).

2. Paired linear spaces and cylinder sets

DEFINITION. *Let E and F be two real linear spaces and let $(x, y) \rightarrow B(x, y)$ be a bilinear form on $E \times F$; we shall say that E and F are paired spaces, by the pairing functional B, if the two following conditions are fulfilled:*

(1) *for every $x \neq 0$ in E, there exists $y \in F$ such that $B(x, y) \neq 0$;*

(2) *for every $y \neq 0$ in F, there exists $x \in E$ such that $B(x, y) \neq 0$.*

For any $y \in F$, let $B_{.y}$ be the linear form $x \rightarrow B(x, y)$ on E, it is clear that $y \rightarrow B_{.y}$ is a linear mapping of F in the algebraic dual space E^* of E. The condition (1) above means that this mapping is injective, and thus it is possible to identify F with its image in E^*, and in the same way E with its image in F^*. When doing so, we write $\langle x, y\rangle$ instead of $B(x, y)$.

DEFINITION. *Let E and F be two linear spaces paired by a bilinear form $(x, y) \rightarrow \langle x, y\rangle$. We call weak topology on E, defined by the duality between E and F, and denote $\sigma(E, F)$, the weakest locally convex topology on E, such that every linear form on E: $x \rightarrow \langle x, y\rangle$, $y \in F$, is continuous.*

The topological dual of E with the topology $\sigma(E, F)$ is F. The weak topology $\sigma(F, E)$ on F is defined in the same way.