ABSTRACT WIENER SPACES

LEONARD GROSS
CORNELL UNIVERSITY

1. Introduction

Advanced integral calculus in infinite dimensions was initiated and developed by R. H. Cameron, W. T. Martin, and their associates in a sequence of papers beginning in 1944.

The underlying space for the integral calculus was the Banach space C consisting of the continuous functions on [0, 1] which vanish at zero. The space C carries the probability measure induced by a one-dimensional Brownian motion. The resulting measure space, generally known as Wiener space, has topological, linear, and measure theoretic structures which are well related to one another for the purposes of analysis over C.

The subset C' consisting of the absolutely continuous functions in C with square integrable derivative forms a Hilbert space with respect to the inner product $(x, y) = \int_0^1 x'(t)y'(t)\,dt$. Here a prime denotes derivative. Although C' is a set of Wiener measure zero, the Euclidean structure of this Hilbert space determines the form of the formulas developed by the above authors, and, to a large extent, also the nature of the hypotheses of their theorems. However, it only became apparent with the work of I. E. Segal [11], [12], dealing with the normal distribution on a real Hilbert space, that the role of the Hilbert space C' was indeed central, and that in so far as analysis on C is concerned, the role of C itself was auxiliary for many of Cameron and Martin's theorems, and in some instances even unnecessary. Thus Segal's theorem ([12], theorem 3) on the transformation of the normal distribution under affine transformations, which is formulated for an arbitrary real Hilbert space H, extends and clarifies the corresponding theorem of Cameron and Martin [1], [2] when H is specialized to C'. This is an extreme case in which consideration of the Banach space structure of C, as opposed to merely the Hilbert space structure of C', contributes little or nothing to a proper understanding of the above theorem. For some other theorems, however, the role of C is not negligible, but nevertheless, it is the relation between C and C' which remains important. Specifically, C is the completion of C' with respect to a norm (the sup norm) on C' which is much weaker than the Hilbert norm on C': $\|x\|^2 = \int_0^1 x'(t)^2\,dt$ and enjoys the property of being a measurable norm on C'.

In this paper we shall abstract this relationship by replacing C' by an arbitrary real separable Hilbert space H and the sup norm by its generalization—

This work was supported by N.S.F. contract G. P. 3754.