ABSTRACT HARMONIC ANALYSIS AND LÉVY'S BROWNIAN MOTION OF SEVERAL PARAMETERS

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1. Introduction

Paul Lévy's studies of the Gaussian process $\{\xi(a), a \in R^d\}$ defined by $E(\xi(a)) = 0$, and

$$E(\xi(a)\xi(b)) = \tfrac{1}{2}(|a| + |b| - |a - b|), \qquad a, b \in R^d, \tag{1.1}$$
$$= f(a, b) \text{ say,}$$

are well known, for example, [12], [13], [14]. He calls this process Brownian motion of several parameters.

Lévy has also studied [15] a Gaussian process $\{\xi(a), a \in S^d\}$ (S^d = the unit sphere in R^{d+1}) defined by $E(\xi(a)) = 0$, and

$$E(\xi(a)\xi(b)) = \tfrac{1}{2}(d(a, o) + d(b, o) - d(a, b)) \tag{1.2}$$
$$= f(a, b) \text{ say,}$$

where $a, b \in S^d$, o is any point (fixed once and for all) of S^d, and $d(x, y)$ stands for the geodesic distance between $x, y \in S^d$, taken along the sphere. This motion may be called Brownian motion with parameter running on S^d [15].

The functions $f(a, b)$ described by (1.1) and (1.2) are both real-valued, symmetric and positive-definite; that is, given $\alpha_1, \cdots, \alpha_n \in R$, $a_1, \cdots, a_n \in R^d$ (or S^d), we have

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i\alpha_j f(a_i, a_j) \geq 0. \tag{1.3}$$

For (1.1) this fact is due to a theorem of Schoenberg [19], and Lévy used this fact to establish the existence of the process ξ. On the other hand, for (1.2), no direct proof of the positive-definiteness is known. The process $\{\xi(a), a \in S^d\}$ is constructed by Lévy by means of white noise integrals, and then it is checked by explicit evaluation that its covariance is (1.2). It follows that (1.2) is positive-definite. Here Lévy was adopting an idea of Chentsov [3], where a white noise integral for $(\xi(a), a \in R^d)$ is described.

The processes described above have several interesting properties, and there seems to be several, as yet not completely clear, connections between their study

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