SIGN-INVARIANT RANDOM ELEMENTS IN TOPOLOGICAL GROUPS

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1. Introduction

The concept of sign-invariant random variables was recently introduced by the writer in [1]: $X_1, \cdots, X_k$ are called sign-invariant if the 2^k joint distributions corresponding to the sets $(\epsilon_1 X_1, \cdots, \epsilon_k X_k)$, $\epsilon_1 \pm 1, \cdots, \epsilon_k = \pm 1$ are all the same. A family of random variables $\{X_t, t \in T\}$, where T is some index set, is called sign-invariant if every finite subfamily consists of sign-invariant random variables. An example is a family of independent random variables with symmetric distributions.

During the last several years, probabilists have been extending their interest from random variables and vectors in Euclidean space to random elements in abstract spaces, particularly topological groups. Grenander's monograph [3] contains a large bibliography of work up to 1963. In this paper we shall generalize some of the properties of sign-invariant random variables on the real line [1] to sign-invariant random elements in a commutative, locally compact topological group G having a countable base. This may be read independently of the previous paper. The definition of sign-invariant random elements is a direct extension of the definition given above for random variables: the X's are elements of the group, and $-X$ is the inverse of X under the group addition.

The Fourier transform was the main tool in the study of sign-invariant random variables on the line; this suggested the generalization to a commutative, locally compact group, which is the natural domain of Fourier analysis.

Section 2 contains the fundamental structure lemmas of sign-invariance. One of the main ones is that sign-invariant random elements are conditionally independent and symmetrically distributed, given a nontrivial sub-σ-field. In section 3 we present some group-theoretic results which characterize the convergence of a sequence of elements in a group G in terms of the convergence of their images under the mappings induced by the character group. The fundamental convergence theorem for series of sign-invariant random elements is given in section 4.

The main part of this paper, sections 5, 6, and 7, is about the stochastic process with sign-invariant increments: a stochastic process in G with a real interval parameter set such that increments of the process over nonoverlapping intervals are sign-invariant. An example is a process with symmetric independent incre-

Research supported in part by National Science Foundation Grant NSF-GP-3694 at Columbia University.