GENERALIZATIONS OF THEOREMS OF CHERNOFF AND SAVAGE ON THE ASYMPTOTIC NORMALITY OF TEST STATISTICS

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1. Introduction

The purpose of the present paper is to generalize the results obtained by Chernoff and Savage [5] on the asymptotic normality of a large class of two-sample nonparametric test statistics.

The assumptions made in [5] involve a certain function J which is assumed to possess two derivatives satisfying boundedness restrictions. However, certain test statistics, for instance those proposed by Ansari and Bradley [1] and Siegel and Tukey [15], do not satisfy the regularity conditions imposed by Chernoff and Savage. In particular, the *first* derivative of the appropriate function J fails to exist at certain points, so that the arguments of Chernoff and Savage are no longer directly applicable.

It will be shown here that the basic asymptotic normality result of [5] remains valid without any assumptions whatsoever or the existence of second derivatives. The assumption of existence of the first derivative is replaced by an assumption of absolute continuity. It should be noted that even this assumption is somewhat too stringent if one is willing to impose restrictions on the couple (F, G). However, the discussion of such possibilities remains beyond the purview of the present paper.

Section 2 of the paper gives a number of definitions which will be used throughout. Section 3 summarizes some properties of the set of functions J which will be used later. The main results are a lemma (lemma 2) on uniform square integrability and a continuity theorem (lemma 3) for the variances of the normal approximations to the distributions of the Chernoff-Savage statistics. Section 4 gives an account of convergence properties of empirical cumulative distributions and of their inverse functions.

The tails of the Chernoff-Savage statistics are bounded in section 5, and the main asymptotic normality theorem appears in section 6. Finally, natural extensions to the c-sample situation are provided in section 7.

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