TOPICS IN RANK-ORDER STATISTICS

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This paper is dedicated to the memory of Dr. Frank Wilcoxon.

1. Introduction

Frank Wilcoxon joined the faculty of the Florida State University in 1960. He brought with him a number of ideas for further research in rank-order statistics and proceeded to develop them in association with colleagues and graduate students. This paper is largely expository and reports research based on his suggestions.

Two major topics are presented: sequential, two-sample, rank tests and multivariate, two-sample, rank procedures.

2. Sequential two-sample rank tests

2.1 *Preliminary remarks.* The two-sample, rank-sum test was introduced by Wilcoxon [29], [30]. Two populations, X- and Y-populations, are given with distribution functions,

$$P(X \leq u) = F(u), \qquad P(Y \leq u) = G(u), \tag{2.1}$$

X and Y being the random variables associated with the two populations. The basic null hypothesis tested is that

$$H_0\colon G(u) \equiv F(u), \tag{2.2}$$

usually with the assumed alternative of location change, $G(u - \theta) \equiv F(u)$.

Samples of independent observations of sizes m and n from X- and Y-populations respectively are taken and ranked in joint array. The sum of ranks, T for the X-sample or S for the Y-sample, is taken as the test statistic, and departures of the statistic from its mean under H_0, $\frac{1}{2}m(m + n + 1)$ or $\frac{1}{2}n(m + n + 1)$, are judged for significance. In order that ties in ranks between X- and Y-observations occur with probability zero, one may restrict F and G to be continuous.

Small-sample tables are available as are approximate, large-sample distributions for the rank sum under H_0. A recent extensive set of tables was developed by Wilcoxon, Katti, and Wilcox [31]. This table is divided into four sections corresponding to four levels of significance, 0.05, 0.025, 0.01, and 0.005 for a

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