HORIZON IN DYNAMIC PROGRAMS

JERZY ŁOŚ
INSTITUTE OF MATHEMATICS OF THE POLISH ACADEMY OF SCIENCES

1. Introduction

The theory of dynamical programs deals with undertaking decisions in time. Usually we have a functional over a set of sequences (or functions), and the task consists in finding a minimum of this functional. The components of the sequences (or the value of the functions—when time is considered to be continuous) represent the decisions, which are to be carried out at the appropriate point of time. As the solution—minimizing the functional—we get a sequence of decisions, which tells us what to do at all future times.

This is a considerable simplification of problems we face in applications. Usually in applications we are not interested in all sequences of decisions, but indeed, we are interested in the particular one which we must carry out at the present stage. However, the functional to be minimized is not completely known to us. This means that many data are needed to define a functional. These data will occur in time, finally allowing selection of one functional from a family of many possible functionals. But when making the first decision, we do not know which one will finally be selected.

In several cases, to compute the optimal first step decision, we do not need all the data of the functional, but only a part of them; for instance, those which will occur up to a specific point of time h in the future. Such a point is called the horizon of the problem. This is the point up to which one has to know the future in order to compute the optimal decision at the present stage.

The idea of horizon goes back to Modigliani, who in [6] and [7] defined it in an intuitive manner. But the ideas of Modigliani were not worked out to a precise form, and therefore, the term "horizon," which may be found in many papers concerned with dynamical programs, is used with various meanings.

In this paper we present a rigorous definition of the notion of horizon. An auxiliary notion is that of a dynamical parameter, which serves to express the information concerning data of the functional occurring in time.

There are two groups of problems basic to the theory of horizon. One of them deals with the properties of solutions computed with the help of a given horizon ("horizonal solutions"); the other one is concerned with the existence of the horizon in specific cases. Since this paper has an introductory character, both groups of problems are represented here, but by weak theorems only.