TWO-ACTION COMPOUND DECISION PROBLEMS

M. V. JOHNS, JR.
STANFORD UNIVERSITY

1. Introduction and summary

The compound decision problem considered here consists of a sequence of component problems, in each of which one of two possible actions must be selected. The loss structure is the same for each component decision problem. Each component problem involves independent identically distributed observations whose common distribution function is unknown but belongs to some specified parametric or nonparametric family of distributions (for example, the family of all Poisson distributions with parameter λ bounded above by some finite number B). This family remains fixed for all component problems. It is assumed that, at the time a decision is made in any particular component problem, the available information includes the data obtained in all previous component decision problems in the sequence.

Compound decision problems of this type arise in situations where routine testing and evaluation programs are in operation. For example, in routine lot by lot acceptance sampling for quality control purposes, each lot of items is sampled, and the lot is either accepted or rejected on the basis of the observations obtained. Another example arises in routine medical diagnosis where a decision between two alternative treatments must be made for each of a continuing sequence of patients on the basis of results obtained from a diagnostic test performed on each patient. In either of these examples records of all past observations could certainly be accumulated.

In the compound decision problem as formulated here, no relationships whatever are assumed to exist among the distributions governing the observations associated with different component decision problems (aside from the requirement that all these distributions are members of a specified general family). A strictly "objective" approach to this situation appears, at first glance, to require that each component problem be treated in isolation with the decision for each problem being based on the observations obtained for that problem alone. It has been known for some time, however, that for certain types of compound decision problems, substantially better performance in terms of average risk incurred for a number of component problems may be obtained by using "compound decision procedures" which make explicit use at each

This research supported in part under Office of Naval Research Contract Nonr-225(53) (NR 042-002).