ON DISCRETE EVASION GAMES WITH A TWO-MOVE INFORMATION LAG

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1. Introduction

This paper deals with extensions of the following game. Although there are other interpretations of this game, we use the traditional one of a ship trying to evade a bomber. This problem is sometimes called the bomber-battleship problem.

A ship is constrained to travel on the integer lattice of the real line. In one time unit, he may move one unit distance to the right or one unit distance to the left. He must move each time; he is not allowed to stay still. A bomber with exactly one bomb flies overhead and wants to drop the bomb on the ship. He may drop the bomb on any point he desires, but it takes two time units for the bomb to fall. He knows that at the end of two units time the ship can only be at one of three places: exactly where it is when he lets go of the bomb, two steps to the right, or two steps to the left. There is no use in dropping the bomb at any but these three points. The ship starts at the origin, and he does not know when or where the bomb is dropped until it hits. The bomber may observe the movements of the ship for as long as he likes before dropping the bomb. He wins one unit from the ship if the bomb hits the ship; otherwise, there is no payoff.

This game-theoretic problem, suggested by Rufus Isaacs, was first solved by Dubins in 1953 and published in [2]. It was solved independently by Isaacs and Karlin [4] using a different method. Further results were obtained by Isaacs [3] and by Karlin [5]. A general theory of games with information lag was studied by Scarf and Shapley [7].

In papers [2] and [4], it is shown that this game has a value $v_1 = (3 - \sqrt{5})/2 = .382\ldots$, that the ship has an optimal strategy which is explicitly described, and that the bomber does not have an optimal strategy (merely ϵ-optimal ones). In other words, there is a strategy for the ship such that no matter what strategy the bomber uses, the ship will not be hit with probability greater than v_1, and, for every $\epsilon > 0$, there is a strategy for the bomber such that no matter what strategy the ship uses, the ship will be hit with probability at least $v_1 - \epsilon$.

A strategy for the ship is a rule telling him at each step with what probability he should go to the right as a function of all his past moves. Such a strategy is said to be *Markov* if this probability depends on the past moves only through

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