ON VALUES ASSOCIATED WITH A STOCHASTIC SEQUENCE

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1. Introduction

Let $\{z_n\}_1^\infty$ be a sequence of random variables with a known joint distribution. We are allowed to observe the z_n sequentially, stopping anywhere we please; the decision to stop with z_n must be a function of $z_1, \cdots, z_n$ only (and not of $z_{n+1}, \cdots$). If we decide to stop with z_n, we are to receive a reward $x_n = f_n(z_1, \cdots, z_n)$ where f_n is a known function for each n. Let t denote any rule which tells us when to stop and for which $E(x_t)$ exists, and let v denote the supremum of $E(x_t)$ over all such t. How can we find the value of v, and what stopping rule will achieve v or come close to it?

2. Definition of the γ_n sequence

We proceed to give a more precise definition of v and associated concepts. *We assume given always*

(a) a probability space $(\Omega, \mathfrak{F}, P)$ with points ω;
(b) a nondecreasing sequence $\{\mathfrak{F}_n\}_1^\infty$ of sub-Borel fields of $\mathfrak{F}$;
(c) a sequence $\{x_n\}_1^\infty$ of random variables $x_n = x_n(\omega)$ such that for each $n \geq 1$, x_n is measurable $(\mathfrak{F}_n)$ and $E(x_n^-) < \infty$.

(In terms of the intuitive background of the first paragraph, $\mathfrak{F}_n$ is the Borel field $\mathfrak{B}(z_1, \cdots, z_n)$ generated by $z_1, \cdots, z_n$. Having served the purpose of defining the $\mathfrak{F}_n$ and x_n, the z_n disappear in the general theory which follows.) Any random variable (r.v.) t with values $1, 2, \cdots$ (not including ∞) such that the event $[t = n]$ (that is, the set of all ω such that $t(\omega) = n$) belongs to $\mathfrak{F}_n$ for each $n \geq 1$, is called a *stopping variable* (s.v.); $x_t = x_{t(\omega)}(\omega)$ is then a r.v. Let C denote the class of all t for which $E(x_t^-) < \infty$. We define the *value* of the stochastic sequence $\{x_n, \mathfrak{F}_n\}_1^\infty$ to be

[1] Research supported in part by National Science Foundation Grant NSF-GP-3694 at Columbia University, Department of Mathematical Statistics.

[2] Research supported by the Office of Naval Research under Contract No. Nonr-266(59), Project No. 042-205. Reproduction in whole or part is permitted for any purpose of the United States Government.