A CLASS OF OPTIMAL STOPPING PROBLEMS

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1. Introduction and summary

Let $x_1, x_2, \cdots$, be independent random variables uniformly distributed on the interval $[0, 1]$. We observe them sequentially, and must stop with some x_i, $1 \leq i < \infty$; the decision whether to stop with any x_i must be a function of the values $x_1, \cdots, x_i$ only. (For a general discussion of optimal stopping problems we refer to [1], [3].) If we stop with x_i we lose the amount $i^\alpha x_i$, where $\alpha \geq 0$ is a given constant. What is the minimal expected loss we can achieve by the proper choice of a stopping rule?

Let C denote the class of all possible stopping rules t; then we wish to evaluate the function

$$v(\alpha) = \inf_{t \in C} E(t^\alpha x_t). \tag{1}$$

If there exists a t in C such that $E(t^\alpha x_t) = v(\alpha)$, we say that t is optimal for that value of α. Let C^N for $N \geq 1$ denote the class of all t in C such that $P[t \leq N] = 1$; then $C^1 \subset C^2 \subset \cdots \subset C$, and hence, defining

$$v^N(\alpha) = \inf_{t \in C^N} E(t^\alpha x_t), \tag{2}$$

we have

$$\tfrac{1}{2} = v^1(\alpha) \geq v^2(\alpha) \geq \cdots \geq v(\alpha) \geq 0. \tag{3}$$

We shall show that as $N \to \infty$,

$$v^N(\alpha) \sim \begin{cases} 2(1-\alpha)/N^{1-\alpha} & \text{for } 0 \leq \alpha < 1, \\ 2/\log N & \text{for } \alpha = 1, \end{cases} \tag{4}$$

from which it follows that

$$v(\alpha) = 0, \qquad \text{for } 0 \leq \alpha \leq 1. \tag{5}$$

(For $\alpha = 0$, J. P. Gilbert and F. Mosteller [4] give the expression $v^N(0) \approx 2/(N + \log(N+1) + 1.767)$; this case is closely related to a problem of optimal selection considered in [2]. It can be shown that $Nv^N(0) \uparrow 2$ as $N \to \infty$.)

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