ON THE ELIMINATION OF NUISANCE PARAMETERS IN STATISTICAL PROBLEMS

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1. Introduction

Consider a family of the distributions $\mathcal{P}_\theta$ characterized by the probability densities $\ell(x, \theta)$ with respect to a dominating measure $\mu(x)$ on the σ_n-algebra of a measurable space $(\mathfrak{X}, \mathfrak{A})$ with a parameter $\theta \in \Omega$. Later on $\mathfrak{X} \subset E_n$ will be a parallelepiped of the n-dimensional Euclidean space, $\mathfrak{A}$ the Borel σ-algebra; $\Omega \subset E_s$, in general, a compact in the s-dimensional Euclidean space, $\ell(x, \theta)$ being a function continuous with respect to θ for a fixed x, and for a given θ—almost everywhere continuous with respect to $\mu(x)$, the Lebesgue measure.

We shall consider the problems of hypothesis testing and unbiased estimation. The first class of problems will be formulated as follows.

Let $\Pi_1(\theta), \cdots, \Pi_r(\theta)$ be continuous functions of $\theta \in \Omega \subset E_s$; $r < s$. The hypothesis H_0 to be tested is composite and consists of the equations

$$\Pi_1(\theta) = 0, \cdots, \Pi_r(\theta) = 0 \tag{1.1}$$

which determine the set Ω_0 in the set Ω.

The alternative H_1 to H_0 consists of the inclusion $\theta \in \Omega \backslash \Omega_0$. Sometimes a Bayes distribution $B(\theta)$ on $\mathrm{y} \backslash \mathrm{y}_0$ is given; this converts H_1 into a simple hypothesis. The last set-up is perhaps not quite natural, but it is convenient for the primary investigation of the composite hypothesis H_0.

We study the tests of H_0 against the alternative H_1.

The problem of unbiased estimation will consist in the investigation of the behavior of the statistics $\xi(x)$ possesing the mathematical expectation $E(\xi|\theta) = F(\theta)$ unbiased with respect to this function in the presence of the relations (1.1).

The way the question is presented above does not, of course, cover all the important problems of hypotheses testing and unbiased estimation. For instance, the problems of sequential analysis are not covered in this way. But in the set-up described above we can find a series of problems which are very interesting and deep from the analytical point of view; some of these will be considered below. The proofs of the theorems formulated below are rather long and complicated; therefore it is not possible to exhibit them in this article. (See [12] for the simplest cases.)