ON PARTIAL PRIOR INFORMATION AND THE PROPERTY OF PARAMETRIC SUFFICIENCY

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1. Summary

The problem of statistical decisions when there is a partial lack of prior information is considered, and a definition of the optimality of a statistical procedure in such a case is given. This optimality is a generalization of both the minimax property and the Bayes property, in the sense that the former property yields optimality in the case of a complete lack of prior information, whereas the latter coincides with the optimality in the case of complete prior information. A characterization of the sufficiency of a sub-σ-field $\mathfrak{B}$ of a σ-field $\mathfrak{A}$ of the parameter space is developed from this point of view. The sufficiency of $\mathfrak{B}$ is defined as the property that a prior distribution on $\mathfrak{B}$ induces the same optimal procedure as a prior distribution on $\mathfrak{A}$. In the case of testing hypotheses, there is shown a connection of this concept with that of the parametric sufficiency due to E. W. Barankin [1].

2. Introduction

For some time there have existed characterizations of the sufficiency of a statistic (or a σ-field in a sample space) from the standpoint of decision functions (see [2], [3], [4], and [5]). According to these characterizations, a statistic $t(x)$ is sufficient if and only if in a certain statistical problem the risk by a decision procedure through the observation of the sample x is not increased at all by the restriction to the observation of the statistic $t(x)$. We shall attempt here to give a parallel discussion in the case of parametric sufficiency, a concept introduced by Barankin [1]. A function $u(\theta)$ on a parameter space Θ is called a sufficient parameter if for any measurable set A the probability $P_\theta(A)$ of occurrence of the observed sample x in A when θ is the true parameter is a function of $u(\theta)$. Looking at "the function on the parameter space" more closely, we understand that this idea represents an amount of prior information. Let us consider this problem by example. Suppose a statistician is informed of nothing but the prior

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