EFFICIENCY IN NORMAL SAMPLES AND TOLERANCE OF EXTREME VALUES FOR SOME ESTIMATES OF LOCATION

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1. Summary

This paper presents a number of separate but interrelated results concerned with estimates for the symmetric one-sample location problem. (1) Devices are discussed which, in the normal case, increase the information obtainable by random sampling experiments by a factor of hundreds or thousands. (2) Using these devices, sampling evidence is presented that supports the asymptotic theory for a recently introduced estimate, here called T. (3) A linear estimate, called W, is proposed as a natural analog of T, and is used to check the sampling experiment. (4) The estimate T is recognized as a member of a class of estimates, and the class is explored for other members that are easier to compute. (5) One of the simplest of these, called D, is seen to correspond to the one-sample analog of Galton's test, whose null distribution is given. (6) The same samples used with T are applied to D, with closely similar results. (7) A simple numerical measure of tolerance to extreme values is proposed, and methods of evaluating it are presented in two classes of cases that cover the estimates here discussed. (8) A number of estimates, including $\overline{X}$, T, D, and the trimmed and Winsorized means, are compared with regard to normal efficiency, ease of computation, and extreme value tolerance.

2. Introduction

Consider the problem of estimating the center μ of a symmetric population on the basis of a sample $X_1, \cdots, X_n$. It was pointed out by Hodges and Lehmann [6] that, in a natural way, an estimate for μ could be formed from any of a class of rank tests of the value of μ. Perhaps the most interesting of the estimates there considered is the one which corresponds to the Wilcoxon one-sample test. This estimate, denoted here by T and defined in the next section, was shown to be asymptotically normal as $n \to \infty$, and to have attractive large-

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