ON BASIC CONCEPTS OF STATISTICS

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1. Summary

This paper is a contribution to current discussions on fundamental concepts, principles, and postulates of statistics. In order to exhibit the basic ideas and attitudes, mathematical niceties are suppressed as much as possible. The heart of the paper lies in definitions, simple theorems, and nontrivial examples. The main issues under analysis are *sufficiency, invariance, similarity, conditionality, likelihood,* and their mutual relations.

Section 2 contains a definition of sufficiency for a subparameter (or sufficiency in the presence of a nuisance parameter), and a criticism of an alternative definition due to A. N. Kolmogorov [11]. In that section, a comparison of the principles of sufficiency in the sense of Blackwell-Girschick [2] and in the sense of A. Birnbaum [1] is added. In theorem 3.5 it is shown that for nuisance parameters introduced by a group of transformations, the sub-σ-field of invariant events is sufficient for the respective subparameter.

Section 4 deals with the notion of similarity in the x-space as well as in the (x, θ)-space, and with related notions such as ancillary and exhaustive statistics. Confidence intervals and fiducial probabilities are shown to involve a postulate of "independence under ignorance."

Sections 5 and 6 are devoted to the principles of conditionality and of likelihood, as formulated by A. Birnbaum [1]. Their equivalence is proved and their strict form is criticized. The two principles deny gains obtainable by mixing strategies, disregarding that, in non-Bayesian conditions, the expected maximum conditional risk is generally larger than the maximum overall risk. Therefore, the notion of "correct" conditioning is introduced, in a general enough way to include the examples given in the literature to support the conditionality principle. It is shown that in correct conditioning the maximum risk equals the expected maximum conditional risk, and that in invariant problems the sub-σ-field of invariant events yields the deepest correct conditioning.

A proper field of application of the likelihood principle is shown to consist of families of experiments, in which the likelihood functions, possibly after a common transformation of the parameter, have approximately normal form with constant variance. Then each observed likelihood function allows computing the risk without reference to the particular experiment.

In section 7, some forms of the Bayesian approach are touched upon, such as those based on diffuse prior densities, or on a family of prior densities.