WEAK LIMITS OF SEQUENCES OF BAYES PROCEDURES IN ESTIMATION THEORY

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1. Introduction

Let $(X, \mathcal{B}, \mu)$ be a totally σ-finite measure space and $\{f(\cdot, \omega), \omega \in \Omega\}$ a family of (generalized) density functions relative to $(X, \mathcal{B}, \mu)$. If $a \in \Omega$ and b is a (randomized) decision procedure for the decision space $\mathfrak{D}$, we borrow Stein's [5] notation and write

$$K(a, b) = \int W(a, t)b(x, dt)f(x, a)\mu(dx), \tag{1.1}$$

where $W(a, t)$ is the measure of loss if $t \in \mathfrak{D}$ is decided and $a \in \Omega$ is the case.

In the sequel we will always suppose that Ω and $\mathfrak{D}$ are locally compact metric spaces and will make suitable measurability assumptions about W, b, f. As is known from the work of Wald [7], under fairly liberal assumptions an admissible procedure b is Bayes in the wide sense. That is, we may find sequences $\{b_n, n \geq 1\}$ and $\{\lambda_n, n \geq 1\}$ such that if $n \geq 1$, b_n is Bayes relative to λ_n, $K(a, b_n) \leq K(a, b)$ for all a in the support of λ_n, and $\lim_{n\to\infty} \int (K(a, b) - K(a, b_n))\lambda_n(da) = 0$. Under convexity assumptions on W one may suppose $b = \text{weak} \lim_{n\to\infty} b_n$, as is explained in the appendix.

If $\mathfrak{D} = (-\infty, \infty)$ and $(\partial W/\partial t)$ is well-defined, then with suitable hypotheses the statement, b_n is Bayes relative to λ_n, is equivalent to the statement, for almost all x, for all t, in the support of $b_n(x, \cdot)$,

$$0 = \int \left(\frac{\partial W}{\partial t}\right)(\omega, t)f(x, \omega)\lambda_n(d\omega). \tag{1.2}$$

If t is vector-valued, (1.2) may be replaced by a system of equations.

Logically, given that b is Bayes in the wide sense relative to $\{b_n, n \geq 1\}$ and $\{\lambda_n, n \geq 1\}$, one would hope to determine a measure $\lambda(\cdot)$ such that for almost all x, for all t in the support of $b(x, \cdot)$,

$$0 = \int \left(\frac{\partial W}{\partial t}\right)(\omega, t)f(x, \omega)\lambda(d\omega). \tag{1.3}$$

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