LIMIT THEOREMS FOR REGRESSIONS WITH UNEQUAL AND DEPENDENT ERRORS

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1. Summary

This paper deals with the asymptotic distribution of the vectorial least squares estimators (LSE) for the parameters in multiple linear regression systems. The regression constants are assumed to be known; the errors are assumed (a) to be independent but not necessarily identically or normally distributed (section 3), or (b) to constitute a generalized linear discrete stochastic process (section 4). The latter part includes the case of regression for time series. Conditions are studied under which the joint distribution functions (d.f.'s) of the vectorial LSE's tend to a multivariate normal d.f. as the sample size increases. In the proof a central limit theorem (CLT) for weighted averages of independent random variables is used. In case (a), a theorem for large classes of linear regressions is proved (theorem 3.2), whose conditions are in a certain sense also necessary. The theorem simultaneously permits consistent estimation of the limiting covariance matrix of the LSE's. The results in case (b) are contained in theorems 4.2, 4.3, 4.4, 4.5, 4.6. They are not naturally of as closed a form as those pertinent to case (a) because of the more complicated nature of the problem. Some use of spectral theory is made. Several examples are discussed (section 3.3). The assumptions made in this paper are weaker than those of results published earlier in the literature. (For a more recent survey, compare [6].) Their structure is quite simple so that they ought to be useful in applications. Section 4.4 contains some remarks on multivariate regression equations.

2. Introduction (notations)

There exists a considerable number of publications dealing with the asymptotic normality of parameter estimates for linear regressions, many of which deal with specific cases, however, or are unnecessarily narrow in the assumptions made. The most general paper among these, and the one closest to

Research supported in part by National Science Foundation Grant NSF-GP-3694 at Columbia University.