ON CONFIDENCE INTERVALS AND SETS FOR VARIOUS STATISTICAL MODELS

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1. Introduction

In addition to obtaining point estimates, one of the central problems of statistical inference is the construction of confidence intervals. In most works, as a rule, considerations are limited to independent sampling, which restricts the range of application without justification. The present paper intends to show that the problems of constructing confidence sets and intervals may be solved for diverse models of mathematical statistics. Underlying the methods is the concept of systems of confidence sets (see [1], [2], [3]). The material expounded below is part of the lectures in a course in mathematical statistics read to students in the Mathematics-Mechanics Faculty of Moscow University in the Fall semester of 1965.

2. Construction of confidence intervals

We consider the statistical model $[X, \mathfrak{B}_X, \Theta, P_\theta]$ where $X = \{x\}$ is the set of possible results x of the experiment, and $\mathfrak{B}_X$ is a σ-algebra of events. A family of stochastic measures P_θ governing the outcome of the experiment is given on $\mathfrak{B}_X$. The object Θ is a set of unknown parameters. Let us consider the subset H of points (θ, x) in the direct product $\Theta \times X$. The sets $H_\theta = \{x\colon (\theta, x) \in H\}$ are called θ-sections of H. The sets $H_x = \{\theta\colon (\theta, x) \in H\} \subseteq \Theta$ are called x-sections of H. The subsets $\{H_x\}$ of the set Θ are called confident with a coefficient of confidence not less than (equal to) γ if the set $\{\theta \in H_x\} \in \mathfrak{B}_X$ and

$$\inf_{\theta \in \Theta} P_\theta\{\theta \in H_x\} \geq (=)\gamma. \tag{1}$$

THEOREM 1. (See [1].) *If the θ-sections H_θ of the set H are measurable, and if for every $\theta \in \Theta$ they satisfy the condition*

$$\inf_{\theta \in \Theta} P_\theta\{x \in H_\theta\} \geq (=)\gamma, \tag{2}$$

then the x-sections of the set $H \subseteq \Theta \times X$ form a system of confidence sets with a coefficient of confidence not less than (equal to) γ, or briefly, a γ system.