PROBLEMS RELATING TO THE EXISTENCE OF MAXIMAL AND MINIMAL ELEMENTS IN SOME FAMILIES OF STATISTICS (SUBFIELDS)

D. BASU
UNIVERSITY OF NORTH CAROLINA
and
INDIAN STATISTICAL INSTITUTE

1. Summary

In statistical theory one comes across various families of statistics (subfields). For each such family, it is of some interest to ask oneself as to whether the family has maximal and/or minimal elements. The author proves here the existence of such elements in a number of cases and leaves the question unsolved in a number of other cases. A number of problems of an allied nature are also discussed.

2. Introduction

Let $(\mathcal{X}, \mathcal{A}, \mathcal{P})$ be a given probability structure (or statistical model). A statistic is a measurable transformation of $(\mathcal{X}, \mathcal{A})$ to some other measurable space. Each such statistic induces, in a natural manner, a subfield (abbreviation for sub-σ-field) of $\mathcal{A}$ and is, indeed, identifiable with the induced subfield.

Between subfields of $\mathcal{A}$ there exists the following natural partial ordering.

DEFINITION 1. *The subfield $\mathcal{A}_1$ is said to be larger than the subfield $\mathcal{A}_2$ if every member of $\mathcal{A}_2$ is also a member of $\mathcal{A}_1$.*

A slightly weaker version of the above partial order is the following.

DEFINITION 2. *The subfield $\mathcal{A}_1$ is said to be essentially larger than the subfield $\mathcal{A}_2$ if every member of $\mathcal{A}_2$ is $\mathcal{P}$-equivalent to some member of $\mathcal{A}_1$.*

As usual, two measurable sets A and B are said to be $\mathcal{P}$-equivalent if their symmetric difference $A \Delta B$ is P-null for each $P \in \mathcal{P}$.

Given a family $\mathcal{F}$ of subfields (statistics), one naturally inquires as to whether $\mathcal{F}$ has a largest and/or least element in the sense of definition 1. In the absence of such elements in $\mathcal{F}$, one may inquire about the possible existence of maximal and/or minimal elements. An element $\mathcal{A}_0$ of $\mathcal{F}$ is a maximal (minimal) element of $\mathcal{F}$, if there exists no other element $\mathcal{A}_1$ in $\mathcal{F}$ such that $\mathcal{A}_1$ is larger (smaller) than $\mathcal{A}_0$. In the absence of maximal (minimal) elements in $\mathcal{F}$, one may look for elements