SOME INEQUALITIES AMONG BINOMIAL AND POISSON PROBABILITIES

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1. Introduction

The binomial probability function

$$b(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad k = 0, 1, \cdots, n, \tag{1.1}$$
$$= 0, \qquad k = n+1, \cdots,$$

can be approximated by the Poisson probability function

$$p(k; \lambda) = e^{-\lambda} \frac{\lambda^k}{k!}, \qquad k = 0, 1, \cdots, \tag{1.2}$$

for $\lambda = np$ if n is sufficiently large relative to λ. Correspondingly, the binomial cumulative distribution function

$$B(k; n, p) = \sum_{j=0}^{k} b(j; n, p), \qquad k = 0, 1, \cdots, \tag{1.3}$$

is approximated by the Poisson cumulative distribution function

$$P(k; \lambda) = \sum_{j=0}^{k} p(j; \lambda), \qquad k = 0, 1, \cdots, \tag{1.4}$$

for $\lambda = np$. In this paper it is shown that the error of approximation of the binomial cumulative distribution function $P(k; np) - B(k; n, p)$ is positive if $k \leq np - np/(n+1)$ and is negative if $np \leq k$. In fact, $B(k; n, \lambda/n)$ is monotonically increasing for all n $(\geq \lambda)$ if $k \leq \lambda - 1$ and for all $n \geq k/(\lambda - k)$ if $\lambda - 1 < k < \lambda$, and is monotonically decreasing for all n $(\geq k)$ if $\lambda \leq k$. Thus

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