INTEGRAL EQUATION DESCRIPTION OF TRANSPORT PHENOMENA IN BIOLOGICAL SYSTEMS

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1. Introduction

In considering transport phenomena in biological systems, usually the purpose is to gain some information about the structure of the system from the analysis of flux and concentration data. In a typical experiment radioactively labeled molecules of some kind are introduced into an animal; subsequently radioactivity of various components of the system is measured, and from this information an attempt is made to characterize the transfer of molecules from one part of the system to another and the chemical conversion of one species to another. Most commonly this characterization has been in terms of a number of compartments or "pools" and "turnover" coefficients between the different pools. A recent review article by Robertson [1] and a forthcoming book by Sheppard [2] survey the literature in this area. Mathematically [3] this approach can be shown to represent an approximation to the basic transport equation

$$-\operatorname{div} \mathbf{J}_k(\mathbf{r}, t) + s_k(\mathbf{r}, t) = \frac{\partial c_k(\mathbf{r}, t)}{\partial t}, \tag{1.1}$$

where $\mathbf{J}_k(\mathbf{r}, t)$ is the total vector flux of particles of the kth species at position $\mathbf{r}$ and time t, $s_k(\mathbf{r}, t)$ is the net production per unit volume from chemical reactions, and $c_k(\mathbf{r}, t)$ is the concentration. Here and subsequently we always refer to the labeled particles unless specifically stated otherwise. Physically this approach can be justified by the existence of various more or less discrete anatomical and physiological compartments and pools in biological systems. It also has the practical justification that frequently flux data take the form of the sum of several exponential terms, which is the form of solution obtained for the compartmental model.

The main difficulty with this approach is that it quite clearly is not a good approximation for certain problems, for example problems in which transport via the circulatory system is important. Such systems can of course be described by the partial differential equation (1.1), but this is essentially vacuous because the equation can rarely be solved for biological geometries. An attempt has been made therefore to find mathematical descriptions more general than the compart-