A MATHEMATICAL MODEL OF THE CHEMISTRY OF THE EXTERNAL RESPIRATORY SYSTEM

GEORGE B. DANTZIG
JAMES C. DE HAVEN
THE RAND CORPORATION
AND
CRAWFORD F. SAMS
UNIVERSITY OF CALIFORNIA, BERKELEY

1. Description of the model

The purpose of this paper is to summarize the mathematical aspects of our more complete study of this subject [1].

From the physiological data available and the working of the respiratory system as outlined in the foregoing reference, we set up a mathematical model to represent the more important of the known interrelated physiological functions and chemical reactions involved in the human respiratory system.

Figure 1 is a schematic illustration of the system and demonstrates the relationship of the inputs and outputs of the mathematical model to the actual system. The a and v in the figure refer to the amounts of the input "elements" or "building blocks" coming to the lungs in unit time from venous blood and from the air. The x refer to the amounts of the resulting numerous molecular species produced in the arterial blood and in the air of the lung sacs when equilibrium is reached and as determined by the solution of the mathematical model. At the present stage outputs of the tissue cells are introduced into the model in terms of the composition of the venous blood.

The model was constructed to provide an accounting for the mass of all the elements involved. Having available the equilibrium constants for the molecules formed, it was possible to establish the thermodynamics of the system within the model. In order to illustrate our approach, we have extracted a small piece of our respiratory model. Let us for simplicity set aside most of the substances found in the blood plasma compartment except for the carbon dioxide dissolved in water. As shown in table I this will result in the forming of chemical species such as H_2O, OH^-, H^+, CO_2, HCO_3^-. We suppose everything is held at constant temperature and pressure and that sufficient time has elapsed for the mixture to settle down. Our problem is to predict the equilibrium distribution.

To build up the chemical equilibrium model we first distinguished the dif-