A MINIMAX ESTIMATOR FOR THE LOGISTIC FUNCTION

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1. Introduction

One of us has discussed the use of the logistic function

$$P_i = 1 - Q_i = \frac{1}{1 + e^{-(\alpha + \beta x_i)}} \tag{1.1}$$

as a model for analyzing bioassay or other experiments with "quantal" response, and has studied the problem of estimating the parameters α and β, in several papers (see [1] and the references given therein). At each of k dose levels $x_1, \cdots, x_k$ we perform n trials. It is assumed that the number R_i of responses among the trials at dose level x_i has the binomial distribution with probability of response P_i given by (1.1), and that all trials are independent. On the basis of the observed values r_i of the random variables R_i we wish to estimate the parameters. The problem also arises in the one-parameter form, where β is taken to be known and only α is to be estimated. It is solely with this one-parameter problem that we are here concerned.

Various estimators for α may be proposed, on the basis of various general principles, such as those of maximum likelihood and minimum chi-square, these having certain desirable asymptotic properties. But realistically, what is of interest is the performance of the estimates for small or moderate sample sizes. In studying the actual performance of the estimates, it was considered necessary to use specific numerical values for the quantities involved. In [1] the mean square errors of several estimators were computed and compared for $n = 10$, $k = 3$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$, and with $\beta = \log(7/3)$. This value of β was chosen so that when $\alpha = 0$ we have $P_1 = 0.3$, $P_2 = 0.5$, $P_3 = 0.7$. Five different values of α were taken, corresponding to $P_2 = 0.5$, 0.6, 0.7, 0.8, and 0.85, and

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