

ON COMPARING SURVIVAL TIMES

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1. Introduction

Let us consider two positive random variables, say $\tau > 0$ and $\tau^* > 0$, representing respectively length of life of individuals, as measured from some suitable starting point $t = 0$, in two hypothetical populations, one characterized by some given set of conditions, say C , the other by a different set of conditions, C^* . We assume that each of the populations is homogeneous so that their members have the same, though unknown, probability of surviving to future times. Let us call

$$(1) \quad P_t = P\{\tau > t\},$$

the survivorship function under C and

$$(2) \quad P_t^* = P\{\tau^* > t\},$$

the survivorship function under C^* .

For example, τ may represent the length of life in a population of children afflicted with nephrosis at approximately the same age who did not receive adrenocortical hormones and τ^* the length of life of similar children who however have been subjected to adrenocortical hormone therapy. Or τ may represent the length of life of tumors implanted in mice when subject to a certain schedule of irradiation, and τ^* represents the regression time of such tumors when the radiotherapy is supplemented by the administration of some antimetabolite.

Our purpose in replacing the set of circumstances C by C^* in both of these examples is to influence the length of life in the population. In the first example, we would like to increase it; in the second, to decrease it. By what criteria should we judge the change, that is, how should the distributions of survival times P_t and P_t^* be compared?

In some branches of medical investigations, notably cancer research, a tradition has been developed by which a fixed value of t is selected, say t_0 and a method of management C^* is judged superior to C if the observations seem to indicate that $P_{t_0}^*$ is larger than P_{t_0} . There is by now a large statistical literature (see, for example, [1] to [17]), supplying methods of estimating and comparing the survivorship functions at a preselected point, to suit different degrees of a priori

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