ON SOME STATISTICAL PROPERTIES OF DYNAMICAL SYSTEMS

S. M. ULAM
UNIVERSITY OF CALIFORNIA
LOS ALAMOS SCIENTIFIC LABORATORY

1. Introduction

It is intended to present in this paper a number of problems and a brief summary of some numerical computations on the asymptotic behavior of certain simple dynamical systems.

These problems refer to the behavior of a few mass points with given mutual interactions and concern the ergodic properties of the system. Broadly speaking, the questions pertain to the time rates with which a statistical or thermodynamical equilibrium-like states, might be attained. That the approach to equilibria as postulated in statistical dynamics might be extremely slow as compared to times obtained by phase-space volumes or relaxation estimates was indicated in some computations performed a number of years ago by J. Pasta, E. Fermi, and the author [1]. This problem dealt with the long time range behavior of a vibrating string with nonlinear forces added to the usual linear ones. In reality, the problem concerned a dynamical system of a finite number (for example, 64) of particles and was pursued numerically over hundreds of cycles, each corresponding to a would-be period, that is, times corresponding to a full period of the purely linear part of the problem. The results were somewhat surprising in that no tendency towards equilibrization of energy between all the possible modes was noted. Instead, these results showed a transfer of energy between the first few modes of oscillation of the string. The high modes (say from number 5 on up to the last), even in their totality do not acquire more than a few per cent of the total potential plus kinetic energy. Ultimately, the system came back practically to the initial condition. An account of this work is also given in my book [2].

Imagine, quite generally, a system of particles with different masses, all considered as mass points which attract each other according to a given law, say with inverse square forces. Let us assume, furthermore, that the system is in a quasi equilibrium in the sense that most of the particles will stay within a certain bounded distance from each other for a time long compared to the time it takes the radius vector of each particle to describe a full rotation through 2π. One question is, how long will it take for the velocities of the particles to be distributed approximately in accordance with the equilibrium law of statistical me-