DENSITY OF PROBABILITY OF PRESENCE OF ELEMENTARY PARTICLES

LAURENT SCHWARTZ
UNIVERSITY OF PARIS

1. Introduction: nonrelativistic case

In the initial, nonrelativistic theory of quantum mechanics it is assumed that the only information we have about the state of a particle, at a given time, is its wave function Ψ, a complex function on R^3 or a complex function of three coordinates x, y, z. This function is assumed to be square integrable, $\Psi \in L^2$, and moreover one assumes

$$\iiint_{R^3} |\Psi(x, y, z)|^2 \, dx \, dy \, dz = 1. \tag{1.1}$$

Consider an observable physical quantity, taking its values in a set X. For example, the position of the particle is a quantity with values in $X = R^3$ and so is the velocity. The energy has values in $X = R$, and so on. In classical mechanics, a measurement of such a quantity is supposed to be obtainable with arbitrary accuracy, and, for a given state, the quantity has a definite value x in X. In quantum mechanics, this unlimited precision disappears. If we make a measurement of the quantity, for a particle having the wave function Ψ, we have only a probability law P_Ψ, depending on Ψ, that is, a positive measure on X, of total mass 1. Thus, if A is a subset of X, assumed to be measurable (P_Ψ), the probability that the measurement will give a result in $A \subset X$ is $P_\Psi(A)$. It is usually assumed that this probability law P_Ψ on X must be given by a spectral decomposition of the Hilbert space L^2, with respect to X. Such a spectral decomposition is defined as follows. It is a map $P: A \to P(A) = L^2_A$, where A runs over a Borel field of subsets of X, and L^2_A is a closed subspace of L^2, with the following properties.

(a) $L^2_\phi = \{0\}$, where ϕ = empty set of X, 0 = origin of the vector space L^2; $L^2_X = L^2$.

(b) If A and B are disjoint subsets of X, L^2_A and L^2_B are orthogonal in L^2.

(c) If A is the union of a finite or denumerable family of disjoint subsets A_n, then L^2_A is the closure of the subspace of L^2 spanned by the $L^2_{A_n}$.

Thus the probability law P_Ψ of the physical quantity under consideration must be given by