SOME PROBLEMS IN THE THEORY OF COMETS, II

DAVID G. KENDALL
MAGDALEN COLLEGE, OXFORD UNIVERSITY

1. Let $y_1, y_2, \cdots$ be independent random variables having the distribution

$$g(y)\,dy, \qquad -\infty < y < \infty, \tag{1.1}$$

where $g(y)$ is an *even* function of y, and let R_m and S_m be defined for $m \geqq 1$ by

$$\begin{aligned} S_m &= y_1 + y_2 + \cdots + y_m, \\ R_m &= \min\,(0, S_1, S_2, \cdots, S_m); \end{aligned} \tag{1.2}$$

let k be a constant in the range $0 \leqq k < 1$. This paper is concerned with the function

$$C(z|x) = \sum_{m=1}^{\infty} (1-k)^m P\{x + R_m > 0, x + S_m \leqq z\}, \tag{1.3}$$

where $x > 0$ and $z \geqq 0$, which we shall study by the methods of Frank Spitzer [6], [7], [8]; the results will then be applied to an astronomical problem formulated in the first part [3] of this paper.

From theorem 4.1 of [6] (or from an earlier theorem of Paul Lévy) we know that lim sup $S_m = +\infty$ and that lim inf $S_m = -\infty$, with probability one, so that infinitely many terms of the sequence

$$x + S_1, x + S_2, \cdots \tag{1.4}$$

will be zero or negative. Let the first such nonpositive term and *all* succeeding terms (of either sign) be removed from (1.4). Let a biased coin show heads with probability k and tails with probability $(1-k)$, and in an infinite sequence of independent throws (independent also of the y) let the first head occur at the Mth throw; we then remove the Mth and *all* subsequent terms from the sequence (1.4) (if they still survive). The quantity $C(z|x)$ defined at (1.3) above will then be the expected number of terms $x + S_m$ in the curtailed sequence which lie in the half-open interval $(0, z]$. It is not clear from this definition that $C(z|x)$ is finite, but this will be proved in due course.

In the astronomical problem $C(z|x)$ is the expected number of complete circuits described round the sun by a comet initially in the positive energy state x,

Formerly Skynner Student in Astronomy, Balliol College, Oxford.