# ON THE DYNAMICAL DISEQUILIBRIUM OF INDIVIDUAL PARTICLES

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In one of the most stimulating papers read at the Fourth Berkeley Symposium, S. Ulam [1] raised the question of how rapidly an assembly of colliding elastic particles would settle down into its equilibrium state. If initially the assembly consists of fast and slow particles, it is a familiar principle that the fast particles will tend to slow down and the slow particles to speed up until, viewed macroscopically, energy is shared uniformly among the assembly. Ulam reported on Monte Carlo studies with a high-speed computer, which tracked the histories of individual particles in such an assembly in regions of one or more dimensions. It was found that the energies of individual particles fluctuated very irregularly, though it might be hoped that, if the computation could have been carried on long enough, individual energies would eventually approach some neighborhood of their expected equilibrium values. The purpose of the present note is to handle theoretically one of the simpler one-dimensional cases, considered by Ulam, and to show that individual energies do *not* approach their equilibrium values: in short, the fluctuations observed on the computer will persist indefinitely, however long the computation. I cannot see any good reason why a similar failure to attain individual equilibrium should not also hold in a real system in three dimensions.

The following discussion also shows that the fluctuations of energy occurring in a fixed region of space are qualitatively different from the fluctuations occurring in a fixed set of particles. The former are less irregular than the latter, but even so do not settle down to equilibrium. The type of disequilibrium treated in the present paper is a persistent instability, to be contrasted with the transient instability discussed at length in the literature. Many papers in this literature assume that instability will be transient and proceed, *on this assumption*, to determine the relaxation times of this transience. For a review of the literature and a bibliography of 157 papers, see [2]; references [3] through [8] provide a selection of subsequent articles.

Ulam considered, among other matters in his talk, the following idealized one-dimensional situation. A weightless elastic particle is constrained to move on a straight line between a reflecting barrier and a heavy particle, which oscillates along the same straight line. Each time the weightless particle strikes the heavy