ON THE STATISTICAL LOSS OF LONG-PERIOD COMETS FROM THE SOLAR SYSTEM. II

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1. Introduction and summary

This paper deals with the distribution of the lifetimes of comets, and the way in which this distribution of lifetimes affects the total population of observable comets regarded as a function of the age of the solar system.

The energy per unit mass of a comet is $-\gamma M/2a$, where γ is the constant of gravitation, M is the mass of the sun, and a is the semimajor axis of the comet's elliptical orbit. In the present work it is convenient to change the sign of this energy, and to work with a quantity $z = c/a$, where c is a positive constant chosen to simplify the notation. The first part of this paper, by R. A. Lyttleton, explained how the value of z is perturbed by Jupiter each time the comet visits the neighborhood of the sun and planets, and how, when successive perturbations eventually lead to a negative or zero value of z, the comet is lost from the solar system along a hyperbolic or parabolic orbit. The lifetime of a comet, brought into the solar system with an energy $z_0 = x$ and having values of z equal to $z_0, z_1, \cdots, z_{T-1}$ at successive orbits up to the moment of loss, is therefore

$$G(x) = V(z_0) + V(z_1) + \cdots + V(z_{T-1}), \tag{1.1}$$

where $V(z)$ is the time taken to describe an orbit with energy value z. Kepler's third law states that $V(z)$ is proportional to $z^{-3/2}$, and this relation is used in various parts of the work. But in other parts of the paper it causes little extra work to treat an arbitrary nonnegative function $V(z)$. One possible advantage of this extra generality is that it will still permit the theory to be applied if, for example, it should turn out that the influence of stellar perturbations upon comets with very long periods can be approximated by some modification of Kepler's third law. Apart from this possibility, however, no account is taken in the present work of the influence of stellar perturbations.

Section 3 of the paper provides the theory for the distribution of the lifetime $G(x)$, when the distribution of successive perturbations $y_t = z_t - z_{t-1}$ has a given arbitrary form $P(y)$. We permit $P(y)$ to be improper at $-\infty$, and thereby allow for losses of comets due to disintegration. Section 2 provides the analogous theory when the perturbations are replaced by a continuous process, namely