THE MOTION OF CHARGED PARTICLES IN A RANDOM MAGNETIC FIELD

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1. Introduction

There are in nature several instances of charged particles with very high energies, distributed over a wide range: cosmic rays, the electrons of the outer Van Allen belt, and protons emitted by the sun in association with flares. The origin of these particles has long presented a puzzle. In 1949, Enrico Fermi pointed out that the motion of charged particles in a randomly changing magnetic field ought to lead, through Faraday's law of induction, to a gradual but unlimited increase in their mean energy [1]. Fermi proposed this mechanism as the source of cosmic rays, the random magnetic field being identified with the field expected to exist in interstellar space, according to Alfvén's ideas on cosmical electrodynamics [2].

If one wishes to study quantitatively the problem of the motion of charged particles in a random magnetic field, one is faced with the obvious difficulty that the equations of motion are complicated and cannot be integrated. It is therefore necessary to introduce certain simplifying assumptions. We shall begin right away by listing the principal assumptions made in this paper, assumptions which are nearly the same as those laid down by Fermi in his treatment of the cosmic ray problem.

2. Outline of the problem

The motion of a particle of charge q and momentum $\vec{p}$ in a magnetic field $\vec{B}$ is associated with a characteristic length $l = 2\pi pc/|q|B$, which is the distance traveled during one cyclotron period (c is the speed of light).

We shall adopt three postulates concerning the nature of the random magnetic field and the charged particles moving in it.

POSTULATE 1. *If L is a characteristic length, suitably defined, associated with the fluctuations of the magnetic field, then $l/L \ll 1$.*

This assumption clearly breaks down for particles of sufficiently large momentum, a restriction which should be kept in mind.

POSTULATE 2. *The magnetic field is embedded in a plasma of infinite electrical conductivity, whose hydrodynamical motion is described by a velocity field $\vec{U}$, small compared with the particle velocity v: $U/v \ll 1$.*