ON A CLASS OF INFINITESIMAL GENERATORS AND THE INTEGRATION PROBLEM OF EVOLUTION EQUATIONS

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1. Introduction

The theory of semigroups of bounded linear operators deals with exponential functions in infinite dimensional function spaces. It has been used, as an operator-theoretical substitute for the Laplace transform method, in the integration problem of temporally homogeneous evolution equations, especially of diffusion equations and wave equations (see Hille and Phillips [5] and Yosida [20], [21]).

The purpose of my paper is to call attention to a class of semigroups which is characterized by either one of the three mutually equivalent conditions to be explained below; one of them reads that the semigroup T_t satisfies

$$\varlimsup_{t \downarrow 0} t \left\| \frac{d}{dt} T_t \right\| < \infty. \tag{1}$$

The semigroups arising from the integration in L_2 of temporally homogeneous diffusion equations belong to this class. And the unique continuation theorem of diffusion equations, inaugurated by Yamabe and Itô [6] may be explained by the time-like analyticity of the corresponding semigroups. The situation has an intimate connection with the theory of analytical vectors published recently by Nelson [14]. There is a procedure to obtain semigroups of our class. Let A be the infinitesimal generator of a contraction semigroup. We can define, following Bochner [3], Feller [4], Phillips [15], and Balakrishnan [1], the fractional powers $-(-A)^{\alpha}$ of A and the semigroups generated by them belong to our class. Balakrishnan gave an interesting application of the operator $-(-A)^{1/2}$ to Hille's reduced Cauchy problem for equations $d^2u/dt^2 + Au = 0$.

Tanabe [19] has recently devised an ingenious method of integration of temporally inhomogeneous evolution equations in Banach spaces: $du/dt = A(t)u$. He assumes, for fixed t, that $A(t)$ is the infinitesimal generator of a semigroup of our class. He further assumes a certain regularity condition with respect to t of $A(t)$ which is the same as that introduced by Kato [8] for the integration of such equations. Under these conditions, Tanabe proved that the solution may be obtained by successive approximation starting with the first approximation $\exp[(t - s)A(s)]$. In this way, he has shown that Levi's classical construction