SECOND-ORDER HOMOGENEOUS RANDOM FIELDS

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1. Introduction

The central fact in the theory of second-order stationary random processes is the existence of the spectral representations of the process $\xi(t)$ as

$$\xi(t) = \int_{-\infty}^{\infty} e^{it\lambda} Z(d\lambda), \tag{1.1}$$

and of the corresponding covariance function $B(\tau) = E\{\xi(t+\tau)\overline{\xi(t)}\}$ as

$$B(\tau) = \int_{-\infty}^{\infty} e^{i\tau\lambda} F(d\lambda). \tag{1.2}$$

Here $Z(\Lambda)$ is a completely additive random set function (random measure), while $F(\Lambda)$ is the usual nonnegative bounded measure on the λ-axis $(-\infty, \infty)$, connected with $Z(\Lambda)$ by the relation

$$F(\Lambda) = E|Z(\Lambda)|^2. \tag{1.3}$$

We assume here that the time parameter t of the process takes on all real values. For discrete parameter random processes the limits of integration in (1.1) and (1.2) must be replaced by $-\pi$ to $+\pi$.

Analogous spectral representations exist for stationary processes with a multidimensional parameter $\mathbf{t} = (t_1, t_2, \cdots, t_n)$, that is, for homogeneous random fields $\xi(\mathbf{t})$ in an n-dimensional space R_n, and for a more general class of homogeneous fields on an arbitrary locally compact commutative group G [see formulas (2.21) to (2.23) below]. Moreover, in the case of a homogeneous field $\xi(\mathbf{t})$ with $\mathbf{t} \in R_n$ any additional assumptions about its symmetry impose special restrictions on the covariance function $B(\boldsymbol{\tau})$ and on the spectral measures $F(\Lambda)$ and $Z(\Lambda)$. From the point of view of applications the most interesting is the case of a homogeneous and isotropic random field, that is, the homogeneous field $\xi(\mathbf{t})$ which possesses spherical symmetry. The general form of the covariance function $B(\tau)$, with $\tau = |\boldsymbol{\tau}|$, of such a field in R_n is given by the well-known formula of I. J. Schoenberg [1], namely

$$B(\tau) = \int_0^{\infty} \frac{J_{(n-2)/2}(\tau\lambda)}{(\tau\lambda)^{(n-2)/2}}\, dG(\lambda), \tag{1.4}$$

where $J_{(n-2)/2}$ is a Bessel function of order $(n-2)/2$, and $G(\lambda)$ is a bounded nondecreasing function. The homogeneous and isotropic random vector fields