REMARKS ON PROCESSES OF CALLS

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1. Introduction

The theory of processes of calls is highly developed. In this paper I am going to consider some questions which, to my mind, have not yet been analyzed sufficiently from the measure theoretic point of view.

Palm [1] dealt with a special kind of conditional probability for stationary processes. Khinchin [2] presented and completed the ideas of Palm. Their methods were simple and elegant but they were of analytical character. In this paper I am going to give a different and, so far as I know, new approach to these problems. I shall confine myself to considering some basic notions and their properties.

As a by-product I have obtained a result from ergodic theory which seems to be of some interest in itself.

2. Discrete time

From the measure theoretic point of view, the theory of stationary processes of calls with discrete time is quite simple and consequently it is not dangerous to omit some technical details. Let us consider a doubly infinite stationary sequence of random variables $\cdots, \xi_{-2}, \xi_{-1}, \xi_0, \xi_1, \xi_2, \cdots$ taking only the value zero or one. It is easy to prove that either there are no calls or they occur infinitely many times in both directions. In symbols

$$(1) \qquad P\{\xi_i \equiv 0 \quad \text{or} \quad \overline{\lim_{i \to +\infty}}\, \xi_i = \overline{\lim_{i \to -\infty}}\, \xi_i = 1\} = 1,$$

where P denotes the probability measure. The first possibility is uninteresting from any point of view. Hence we may suppose that

$$(2) \qquad P\{\xi_i \equiv 0\} = 0.$$

Further, the general case can be reduced to this case by the introduction of a "new" probability measure, invariant under the shift transformation,

$$(3) \qquad P^*(\cdot) \overset{df}{=} P(\cdot|N), \qquad N = \{\xi_i \equiv 0\}.$$