AN APPLICATION OF THE CENTRAL LIMIT THEOREM

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1. Introduction

The limit theorems established for the classical case of sums of independent quantities were not adequate for those questions which arose both in the theory of probability itself and in its applications.

As far back as the time of Bernstein's work [1], attempts were made to extend these theorems to the case of dependent quantities. The most definitive results [2] to [5] in this direction were, of course, obtained for quantities connected in a Markov chain.

It is abundantly clear that it would be desirable to establish some general limit theorems at least for quantities which are, in some sense, weakly dependent. The concept of m-dependent random quantities, to which the results of the classical case of independent quantities can be fairly easily generalized [6] to [8], arose in a natural way.

Recently somewhat different conditions for weak dependence appeared, the use of which led to the establishment [9] to [11] of a series of new limit theorems. By far the widest of these conditions was formulated by Rosenblatt [9] for the case of a stationary sequence $\xi(t)$. It consists of the requirement that

$$|P(AB) - P(A)P(B)| \leqq \alpha(\tau), \tag{1}$$

where $A \in M^{t}_{-\infty}$, $B \in M^{\infty}_{t+\tau}$ and M^t_s is the σ-algebra generated by the events of the form $\{\xi(u) < x\}$ for $s \leqq u \leqq t$ and $\alpha(\tau) \rightarrow 0$ when $\tau \rightarrow \infty$.

In this form condition (1), which, following Rosenblatt [9], we shall call the *strong mixing condition*, was applied to the arbitrary random process $\xi(t)$ and generally to some family of σ-algebras M^t_s of ω-sets in a space Ω with a probability measure $P(d\omega)$.

The strong mixing condition (1) is satisfied in the broad class of ergodic Markov processes and also in Gaussian processes. In [12] it was established that for a stationary Gaussian process the strong mixing property is associated with the smoothness of its spectral density; for example, in the case of discrete time it is always satisfied if the spectral density is continuous and never vanishes.

For quantities $\xi(t)$ satisfying the strong mixing condition (1), the central limit theorem itself was obtained in [11] together with more precise details [17]