INDEPENDENCE AND DEPENDENCE

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1. Introduction

A stochastic process is commonly used as a model in studying the behavior of a random system through time. It will be convenient for us to take the stochastic process $\{x_t\}$ as *discrete in time* $t = \cdots, -1, 0, 1, \cdots$. Processes of independent random variables are the simplest and most completely understood. It is, however, clear that these are extremely limited in scope as models and one must have recourse to dependent processes (the random variables x_t not independent) in order to have any power in description. For simplicity, *let us further restrict ourselves to* processes that are stable through time, *stationary processes*. For such processes the probabilities of events shifted through time remain the same, that is, the probability

$$P\{x_{t_1+h} \leqq a_1, \cdots, x_{t_n+h} \leqq a_n\} \tag{1}$$

is independent of h. Such models occur fairly often in the physical sciences. If mean properties of the process are to be capable of being estimated reasonably well from part of a realization of the process, some form of asymptotic independence for blocks of random variables of the process that are widely separated must be satisfied. This is, in effect, the gist of many of the results in ergodic theory. Two types of interesting problems are posed. *The first* of these is *concerned with reasonable notions of asymptotic independence* and *what types of processes satisfy them. The second is that of characterizing those processes* $\{x_t\}$ *that can be constructed out of independent processes by a function and its shifts*, that is,

$$x_t = f(\cdots, \xi_{t-1}, \xi_t, \xi_{t+1}, \cdots) \tag{2}$$

where $\{\xi_t\}$ is a process of independent random variables. Neither of these questions have elicited satisfactory answers. However, there are some small results that do give insights into the problems. The object of this paper is a presentation and discussion of a few of these limited results.

2. Mixing

Ergodicity itself might be thought of as a form of asymptotic independence. However, the most obvious formulation of asymptotic independence is the

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