COMPETITION PROCESSES

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1. Introduction

Some of the simpler theoretical models which have been proposed for phenomena (for example, the competition between species or the occurrence of epidemics) which involve stochastic interactions between several populations have the common feature that they are Markov processes, homogeneous in time, with a countable set of states (m, n) where m and n represent the sizes of two populations. These processes are specified by prescribing the rates at which transitions occur, only transitions to "neighboring" states being allowed: a formal definition of such "competition processes" will be given in section 2.

It is usually difficult to find explicit formulas for the transition probabilities $p_{ij}(t)$ of a Markov process, or even for their limiting values π_{ij} as $t \to \infty$, when the process is defined in terms of the transition rates. However, there are simpler questions worth an answer, relating to recurrence and mean recurrence times and, if there are absorbing states, to absorption probabilities and mean absorption times. We shall discuss such problems for competition processes.

2. Definitions and statement of results

2.1. We consider a time-homogeneous Markov process with a countable set of states $i, j, k, \cdots$, continuous time parameter t, and transition matrix $\{p_{ij}(t)\}$. The process will be specified by prescribing the transition rates

$$q_{ij} = p'_{ij}(0) \tag{1}$$

subject to the conditions

$$\begin{aligned} q_{ij} &\geqq 0, \qquad i \neq j, \\ -q_{ii} &= q_i \geqq 0, \\ \sum_{j \neq i} q_{ij} &= q_i < \infty. \end{aligned} \tag{2}$$

At least one such transition matrix exists; if there is exactly one, we call the set $Q = \{q_{ij}\}$ and the unique associated transition matrix regular. (Thus regularity means that the prescribed transition rates do in fact specify the process: see [7] for further discussion.)

Suppose now that the states are labeled (m, n) where $m, n = 0, 1, 2, \cdots$, and that Q has the structure