# THE METHOD OF CHARACTERISTIC FUNCTIONALS

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## 1. Introduction

The infinite-dimensional analogue of the concept of characteristic function, namely the characteristic functional (ch.f.), was first introduced by A. N. Kolmogorov as far back as 1935 [31], for the case of distributions in Banach space. This remained an isolated piece of work for a long time. Only in the last decade have characteristic functionals again attracted the attention of mathematicians.

Among the works devoted to this subject I shall note here in the first place those of E. Mourier and R. Fortet (see for instance [14], [15], [32], where further references are given) and the fundamental investigations of L. Le Cam [1]. The special case of distributions in Hilbert space is considered in detail, for instance, in the author's work [22].

Let $X$ be a linear space and let $\mathfrak{T}$ be a locally convex topology in this space, let $X^*$ be the space dual to $(X, \mathfrak{T})$, that is, the linear space whose elements $x^*$ are continuous linear functionals over $(X, \mathfrak{T})$. It is quite natural that the following two questions occupy a central position in the general theory. In the first place, when is a nonnegative definite function $\chi(x^*)$ a ch.f. of some $\sigma$-additive measure? Secondly, how can the conditions of weak convergence of distributions be expressed in terms of ch.f.? The content of this article is, in fact, connected with these two questions.

Sections 2 and 3 are of auxiliary character. In section 2 are given some facts about measures in completely regular spaces. Here is introduced the notion of tightness of the measure, which is one of the fundamental concepts of the whole theory.

Section 3 contains an enumeration of some needed results from the theory of locally convex spaces. Particular attention is given to spaces that are the dual of Fréchet spaces, since the latter possess many "good" properties from the point of view of this theory.

In section 4 there is introduced the concept of a weak distribution $P$ in a linear topological space $(X, \mathfrak{T})$. Roughly speaking, the problem is as follows. In every "real" distribution in $X$ the linear functionals become random variables and the joint distribution of any finite number of them