ASYMMETRIC ORIENTED PERCOLATION ON A PLANE

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1. Introduction

The percolation process considered is over the lattice of integer points (x, y), the bonds (all of unit length) being parallel to the x- and y-axes and positively oriented. The "x" bonds have probability $1 - p$ and the "y" bonds probability $1 - p'$ of being dammed. Estimates are given for "critical" pairs of values p, p'.

The terminology used is that introduced by S. R. Broadbent and J. M. Hammersley in [1] and the medium here considered is a crystal in the sense defined there. Its atoms are the points (x, y), where x and y are integers, while its bonds are of two types, dammed with probability $q = 1 - p$, $q' = 1 - p'$ respectively, namely

$$(1) \qquad \begin{aligned} &(x, y) \to (x + 1, y) &\quad &\text{with } q = 1 - p, \\ &(x, y) \to (x, y + 1) &\quad &\text{with } q' = 1 - p'. \end{aligned}$$

Given a single wet atom as source, and the general principle that fluid flows from a wet atom along an undammed oriented bond to wet another atom, we now consider the

PROBLEM. *For what values of p and p' is it true that, with probability* 1, *only a finite number of atoms in all will be wet?*

We shall see that the answer to this question is unchanged if the source consists of any finite number of wet atoms. We shall further see that there exists a curve passing through the points $(0, 1)$ and $(1, 0)$ such that if the point (p, p') lies inside the curve [that is, on the same side as the origin $(0, 0)$] then, with probability 1, only a finite number of atoms will be wet, whereas, if (p, p') lies outside the curve, there is a nonzero probability that an infinity of atoms will be wet.

Hammersley [2], [3] has given lower and upper bounds for the value of p where the line $p = p'$ meets this curve. Estimates are here given for the values of ρ where the line $p = \rho \cos \theta$, $p' = \rho \sin \theta$ meets the curve, for all values of $\theta = 0, \pi/32, \cdots, \pi/2$. This is believed to be the first published example of a percolation process in which the bonds have not all the same probability of being dammed.

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