RECENT DEVELOPMENTS IN THE THEORY OF CHARACTERISTIC FUNCTIONS

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1. Introduction

The study of characteristic functions has several aspects. Characteristic functions were introduced to permit the application of powerful analytical methods in probability theory. They were first used as a tool to study limit theorems, however the scope of their applications has constantly widened and includes now a large variety of problems in probability theory and in mathematical statistics. More recently mathematicians began to investigate problems concerning characteristic functions for their intrinsic mathematical interest. In some of these problems their probabilistic or statistical origin is still apparent, in others, the analytical character becomes dominant.

The present survey deals, therefore, with a variety of loosely connected topics. In the first part, we study certain frequency functions whose characteristic functions are known. These include the stable distributions, Pólya type distributions, and a related family. These results are interesting from the probabilistic as well as from the analytic viewpoint. The second part deals with problems motivated by certain statistical questions. The third part treats the arithmetic of distribution functions and related analytical problems.

Let $F(x)$ be a distribution function, that is a nonnegative, right-continuous function such that $F(-\infty) = 0$ while $F(+\infty) = 1$. The Fourier-Stieltjes transform of $F(x)$, that is the function

$$f(t) = \int_{-\infty}^{\infty} e^{itx}\, dF(x) \tag{1.1}$$

is called the characteristic function of $F(x)$. In this paper we denote distribution functions by capital letters, as $F(x)$, and the characteristic function of $F(x)$ by the corresponding small letter, as $f(t)$. If subscripts are used on the symbol for a distribution function then the same subscripts are attached to its characteristic function.

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