ON THE PROBABILITY OF LARGE DEVIATIONS FOR THE SUMS OF INDEPENDENT VARIABLES

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1. Introduction

The classical theory of the summation of independent random variables as expounded in the book [8] in its simplest case considers the increasing sums $S_n = X_1 + \cdots + X_n$. For the properly normed and centered sums $Z_n = S_n/B_n - A_n$ the behavior in the limit of the probability measures generated by $\{Z_n\}$ on the real axis is studied.

The most general theorems are the integral theorems on the limit behavior of

$$P\{Z_n < x\}. \tag{1.1}$$

Although the theory of local limit theorems is rather well developed [8], it is not yet of such finished character as that of integral limit theorems. Limit theorems for the expression (1.1) usually suppose that $n \to \infty$ and x is a fixed number.

However, many problems occurring in such different fields as mathematical statistics [4], [2], information theory [5], [19], statistical physics of polymers [18], rubber chemistry [17], and even analytical arithmetics [11] require certain information on the limit behavior of (1.1) not contained in the classical limit theorems. The information required concerns the asymptotic behavior of

$$P\{Z_n > x\} \tag{1.2}$$

for "large values" of x, that is for $x = x_n$ increasing as n increases; the corresponding problems will be called problems on the probability of large deviations. As probabilities of events of this kind are generally small, in general, the usual methods of establishing the limit theorems (characteristic functions, partial differential equations) are too rough to give satisfactorily general results and the desired asymptotic results were considered in the literature under certain very stringent conditions imposed upon the variables X_j.

The first theorem on the probability of large deviations was published by A. I. Khinchin [10] in 1929 and related to the particular case of the Bernoulli variables. The same case was treated more completely by N. V. Smirnov [16]. In 1938 appeared the fundamental paper [4] of H. Cramér containing the first result of a general nature in the theory of large deviations. It was improved by W. Feller [7] and by V. V. Petrov [12].