VECTOR-VALUED PROCESSES

HENRY HELSON[1]
UNIVERSITY OF CALIFORNIA, BERKELEY

AND

DAVID LOWDENSLAGER[2]
PRINCETON UNIVERSITY

1. Introduction

The theory of prediction for second order stationary stochastic processes has taken an established place in probability theory, as a result of the complete and elegant treatment achieved by successive authors [2], [3], [4], [6], [12], [13]. More recently the generalization to *multivariate processes* (consisting of several separate but correlated processes with correlations stationary in time) has been taken up by several authors [5], [8], [9], [10], [11], following the beginning made by Wold in this direction. In this paper we survey this problem of multivariate prediction, adopting our own rather eccentric point of view but drawing on the work of others (especially of Masani and Wiener) without hesitation.

2. The prediction problem

Let $y(t)$, with $t = 0, \pm 1, \pm 2, \cdots$, be a vector-valued stochastic process; that is, a random sequence of column vectors, whose components $y^j(t), j = 1, 2, \cdots, N$ are complex random variables. We assume the process is *stationary* in the sense that the inner products

$$(1) \qquad (y^j(t), y^k(s)) = E[y^j(t)\bar{y}^k(s)] = R^{jk}(t - s); \qquad j, k = 1, \cdots, N$$

depend only on the difference $t - s$ of the time arguments. The sequence of matrices $R(t)$, t integral, whose elements are $R^{jk}(t)$ is called the *covariance sequence* of the process. There is a Borel measure, called the *spectral measure* of the process, whose values are positive semidefinite matrices of order N, and whose Fourier-Stieltjes coefficients are the matrices $R(t)$,

$$(2) \qquad R(t) = \int_0^{2\pi} e^{-it\theta}\, dM(e^{i\theta}).$$

(Such matrix or vector equations and integrals can be easily interpreted in terms of the components involved.) For example, if the process is *orthonormal*, which means that each $y^j(t)$ has norm one and is orthogonal to $y^k(s)$ unless $j = k$ and

[1] Supported in part by the National Science Foundation.
[2] Supported in part by the Research and Development Command of the U.S. Air Force.