ASYMPTOTIC EXPANSIONS IN PROBABILITY THEORY

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1. Introduction

The natural solution to many problems in probability theory and mathematical statistics is provided by an examination of certain limit distributions. As is the case in the classical problem of the summation of a large number of random variables which are in some sense of equal weight, the study of the exact distribution functions of the sums not only leads, as a rule, to intractable formulas but in many important cases is impossible, since the exact distributions of the separate summands are often unknown. On the other hand, limit distributions are almost independent of the idiosyncrasies of the distributions of the summands and have a quite manageable form. The same phenomenon can be observed in mathematical statistics in the study of statistical criteria for a large number of observations. As a rule, the exact criteria are complicated but their limiting form is simple and convenient for application. An excellent example of this situation is the treatment of statistical mechanics by A. I. Khinchin, in which limit theorems play an outstanding role. Examples of this kind are literally innumerable.

However, as was pointed out a long time ago by P. L. Chebyshev [1], in order to be able to apply limit theorems in practice, it is necessary to have an estimate of the error involved. Obviously, if the remainder terms decrease slowly, then the limiting distributions must be used with corrections. Currently the most powerful and general method for finding corrections of this nature is the method of asymptotic expansions. These expansions were first examined, without an exact foundation, by Chebyshev [2] for the case of the classical limit theorem. Later, expansions of Chebyshev's type were studied by H. Bruns [3], C. Charlier [4], and F. Y. Edgeworth [5]. However, the fullest results in this direction were obtained much later by H. Cramér [6] and by C. G. Esseen [7]. We shall not concern ourselves here with the numerous further improvements in precision, since that is not the basic object of this paper. Nor shall we dwell on the series of interesting investigations of recent years, which studied asymptotic distributions for the sums of random variables and vectors forming a simple Markov chain. To a considerable extent, these works repeated methods which had been worked out for the sums of independent random variables and essentially consisted of the use of the properties of the Fourier transforms of functions of bounded variation.

The asymptotic analysis of the distributions of functionals of sums of random