CHARACTERIZATION OF SAMPLE FUNCTIONS OF STOCHASTIC PROCESSES BY SOME ABSOLUTE PROBABILITIES

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1. Introduction

The investigation of properties of sample functions of different stochastic processes has attracted much attention. It seems, indeed, that the characterization of a stochastic process is not exhaustive unless such basic properties of sample functions as continuity, kind of discontinuity (if any), integrability, and so on, are known. The most advanced investigations in this field are for certain particularly distinguished classes of stochastic processes, namely Markov processes, processes with independent increments, and martingales. There are important results due to Doeblin, Doob, Lévy, Wiener, and others. For arbitrary stochastic processes, without the assumption that they belong to some traditionally distinguished class of stochastic processes, conditions have been given, expressed in terms of the moments of the random variables of the processes considered, under which almost all sample functions are continuous (Kolmogorov, [11]) or have no discontinuities of the second kind [12]. The author [7] has given conditions expressed in terms of some absolute probabilities under which almost all sample functions of the process are jump functions with a finite expected number of discontinuities. The object of this note is to strengthen these results and to obtain other related results.

2. Notation and summary

We consider the real separable (see p. 51, [4]) stochastic process $\{x_t, t \in I_0\}$ where I_0 is a closed interval. We denote by Ω the set of elementary events ω, by $\mathcal{B}$ the smallest Borel field of ω-sets with respect to which all the random variables x_t where $t \in I_0$, are measurable, and by P the probability measure on $\mathcal{B}$. As is known [9], the Borel field $\mathcal{B}$ is generated by the aggregate of ω-sets of the