TRANSFORMATIONS OF MARKOV PROCESSES CONNECTED WITH ADDITIVE FUNCTIONALS

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1. Introduction

Direct probabilistic constructions that allow us to build one Markov process from another are of interest in the theory of Markov processes as well as in a number of problems in mathematical analysis. In fact, if a Markov process $\tilde{X}$ can be obtained by means of a sufficiently simple transformation of a Markov process X, then it is possible to derive properties of the trajectory of the process $\tilde{X}$ from those of the trajectory of the process X. On the other hand, the solution of many problems in the theory of differential equations, as well as more general operator equations, can be expressed by actual formulas in terms of probability distributions connected with Markov processes. Therefore, by making use of transformations of Markov processes, it is possible to reduce problems of this type for more complicated operators to analogous problems for simpler operators.

In the present paper a general class of transformations of Markov processes is introduced and discussed whose brief description (for stationary Markov processes) is contained in the survey article [5] and in the note [8]. This class of transformations includes as special cases a number of special transformations that were discussed earlier, such as the formation of subprocesses [4], the transformation of the Wiener process which produces a drift (see for example [9]), and others. In the construction of this general class of transformations an important role is played by the concept of an *additive functional* of a Markov process. An additive functional of a Markov process X is a collection of random variables φ_t^s with $s \leqq t$ having the following two properties: (a) φ_t^s is defined in terms of the process in the time $[s, t]$ (a more exact formulation of this condition is given in 2.1A) and (b) $\varphi_t^s + \varphi_u^t = \varphi_u^s$ for all $s \leqq t \leqq u$.

The main results of the paper are given in sections 4 through 6 while section 2 is of an introductory nature. In it are given fundamental definitions and notations of the theory of Markov processes following the monograph [4]. In section 3 are given definitions and examples of additive functionals and some other allied subjects (multiplicative functionals, almost additive functionals, and so forth). The general construction giving transformations of Markov processes is described in sections 4 and 5. In section 6 conditions are studied under which the homo-