A STOCHASTIC TREATMENT OF SOME CLASSICAL INTERPOLATION PROBLEMS

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1. The nonstochastic theory

The interpolation problems to be discussed in this paper arise in the theory of approximation by rational functions in the complex domain. The problems are connected with the following basic question, here stated rather informally. Let B denote a bounded point set of the complex z-plane, let the function f be given on the boundary of B, and let $\{S_n : z_{1n}, z_{2n}, \cdots, z_{nn}\}$ be a sequence of point sets chosen somehow on the boundary of B. Let $L_n = L_n(f; z)$ denote the polynomial in z of degree at most $n - 1$ which is found by interpolation to the values of f at the points S_n. Under what circumstances will $\lim_{n\to\infty} L_n$ exist, and when it does what will the limit be? Preferably of course it will be related in some way to f.

If B is the unit disk, the question becomes one of a special kind of trigonometric interpolation, but not of a type which has been studied intensively as such. If B is the real interval $-1 \leqq z \leqq 1$ the question involves interpolation by real polynomials, or trigonometric interpolation by cosine polynomials. Such problems have been thoroughly investigated over the last fifty years (see Zygmund [1], chapter 10). Attention in the general complex case has been centered on convergence at interior points of B rather than on the boundary of B where the interpolation points are placed, and the required techniques appear to be quite different from those useful in the purely real case. It is the complex case with which this paper is solely concerned.

The history of the complex case might be said to go back to Méray [2], who in 1884 came up with a slightly disturbing example. He pointed out that if B is the unit disk, and S_n consists of the nth roots of unity, and $f(z) = 1/z$, then $L_n(f; z) = z^{n-1}$. This L_n has the limit zero for $|z| < 1$, and elsewhere diverges except at $z = 1$, where it equals the corresponding value of f for all n. Except at $z = 1$ the limit, where it exists, seems to bear little relation to the function to which L_n interpolates. However, later work showed that if the boundary of B consists of one or more rectifiable curves, then what one should be looking for is convergence to the function

This research was supported by the U.S. Air Force through the Air Force Office of Scientific Research of the Air Research and Development Command, under Contract No. AF 49(638)-862.