ON SOME CLASSES OF NONSTATIONARY STOCHASTIC PROCESSES

HARALD CRAMÉR
STOCKHOLM

1. Introduction

This paper will be concerned with stochastic processes with finite second-order moments. We start from a given probability space $(\Omega, \mathfrak{F}, \mathfrak{P})$, where Ω is a space of points ω, while $\mathfrak{F}$ is a Borel field of sets in Ω, and $\mathfrak{P}$ is a probability measure defined on sets of $\mathfrak{F}$.

Any $\mathfrak{F}$-measurable complex-valued function $X = x(\omega)$ defined for all $\omega \in \Omega$ will be denoted as a *random variable*. We shall always assume that

$$\begin{aligned} Ex &= \int_\Omega x(\omega)\, d\mathfrak{P} = 0, \\ E|x|^2 &= \int_\Omega |x(\omega)|^2\, d\mathfrak{P} < \infty. \end{aligned} \tag{1}$$

Two random variables which are equal except on a null set with respect to $\mathfrak{P}$ will be regarded as identical, and equations containing random variables are always to be understood in this sense.

A family of random variables $x(t) = x(t, \omega)$, defined for all t belonging to some given set T, will be called a *stochastic process* defined on T. With respect to T, we shall consider only two cases:

(i) T is the set of all integers $n = 0, \pm 1, \pm 2, \cdots$,

(ii) T is the set of all real numbers t.

With the usual terminology borrowed from the applications, we shall in these cases talk respectively of a stochastic process with *discrete time*, or with *continuous time*. In the first case, where we are concerned with a sequence of random variables, we shall usually write x_n in place of $x(n)$.

With due modifications, the majority of our considerations may be extended to cases where T is some other set of real numbers.

We shall also consider *finite-dimensional vector-valued stochastic processes*, writing

$$\mathbf{x}(t) = \{x^{(1)}(t), x^{(2)}(t), \cdots, x^{(q)}(t)\}, \tag{2}$$

where $\mathbf{x}(t)$ is a q-dimensional column vector, while the components $x^{(1)}(t), \cdots, x^{(q)}(t)$ are stochastic processes in the above sense.