PROBABILISTIC METHODS IN MARKOV CHAINS

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1. Introduction

To avoid constant repetition of qualifying phrases, we agree on the following notation, terminology, and conventions, unless otherwise specified.

$\mathbf{I}$ is a denumerable set of indices. The letters i, j, k, and l, with or without subscript, denote elements of $\mathbf{I}$.

$\bar{\mathbf{I}} = \mathbf{I} \cup \{\infty\}$ is the one-point compactification of $\mathbf{I}$ considered as an isolated set of real numbers; $\infty > i$.

$\mathbf{N}$ is the set of nonnegative integers used as ordinals. The letters ν and n denote elements of $\mathbf{N}$.

$\mathbf{T} = [0, \infty)$; $\mathbf{T}^0 = (0, \infty)$. The letters s, t and u, with or without subscript, denote elements of $\mathbf{T}^0$.

A statement or formula involving an unspecified element of $\mathbf{I}$ or $\mathbf{T}^0$ is meant to stand for every such element.

A sequence like $\{f_i\}$ is indexed by $\mathbf{I}$; a matrix like (p_{ij}) is indexed by $\mathbf{I} \times \mathbf{I}$; a sum like $\sum_j$ is over $\mathbf{I}$.

A function is real and finite valued. A function defined on $\mathbf{T}^0$ and having a right hand limit at zero is thereby extended to $\mathbf{T}$; if in addition it is continuous in $\mathbf{T}^0$ it is said to be continuous in $\mathbf{T}$.

A (*standard*) *transition matrix* is a matrix (p_{ij}) of functions on $\mathbf{T}^0$ satisfying the following conditions:

$$p_{ij}(t) \geqq 0, \tag{1.1}$$

$$\sum_j p_{ij}(t) p_{jk}(s) = p_{ik}(t + s), \tag{1.2}$$

$$\lim_{t \downarrow 0} p_{ii}(t) = 1, \tag{1.3}$$

$$\sum_j p_{ij}(t) = 1. \tag{1.4}$$

A (*temporally*) *homogeneous Markov chain*, or a *Markov chain with stationary transition probabilities*, associated with $\mathbf{I}$ and (p_{ij}), is a stochastic process $\{x_t\}$, $t \in \mathbf{T}$ or $t \in \mathbf{T}^0$, on the probability triple $(\Omega, \mathfrak{F}, \mathbf{P})$, with the generic sample point ω, having the following properties:

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