APPRECIATION OF KHINCHIN

J. L. DOOB
UNIVERSITY OF ILLINOIS

Probability students can hardly visualize the state of probability theory in the nineteen twenties. There were numerous texts on the subject, almost all of which could as well have been written a century earlier, since they were largely collections of elementary combinatorial problems without a unified point of view, together with perhaps some unorganized material on "continuous probabilities." Their authors did not treat probability theory as a mathematical subject, but as the analysis of certain more or less practical problems. One of the better books contained a "proof" that no such theorem as what is now called the strong law of large numbers could be valid for independent random variables with a common distribution! Reputable statisticians were not sure of the relation between independence and orthogonality of random variables, in particular whether orthogonality implies independence. The place of probability theory was so low that one prominent statistician remarked that he supposed it was possible to teach probability apart from statistics, but that doing so would be a tour de force in which he could see no point.

The mathematical background of probability was confused even in the periodical literature. It was not yet clear that there was a distinction between probability as a mathematical theory and as a theory of real events. This confusion enlivened meetings with heated controversies, unhappily absent now that probability has lost its youthful charm and vagueness.

A change was imminent, however. Borel's discussion of "denumerable probabilities" in 1909 had attracted attention to a new class of problems, those involving complete additivity, and Lebesgue's measure theory had already provided the needed mathematical background. It was becoming clear that the connection between probability and measure theory was at least very close. In certain special cases at least, for example in Wiener's discussion of Brownian motion in 1923, studies of infinite collections of random variables were carried out by representing the random variables as measurable functions on a measure space.

The turning point was the appearance of Kolmogorov's monograph in 1933, which laid the basis for probability in terms of measure theory. The Russian mathematical group was at a great advantage in that they were the heirs of a strong tradition, going back to the work of Chebycheff and A. A. Markov and continuing with Bernstein. Unfortunately the significance of the Russian work was misunderstood or ignored until about the middle thirties. One of the leaders of the new Russian school, whose span of mathematical activity covered the