

# ON TWO-SAMPLE TESTS BASED ON ORDER STATISTICS

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## 1. Introduction

1.1. The present paper contains some remarks on the comparison of two samples in one and in two dimensions based on order statistics. The aim is to point out some possibilities for refinement of two-sample tests and to obtain statistics whose exact distribution can be calculated easily.

1.2. Tests based on order statistics have advantages and disadvantages over tests which utilize the specific form of the distributions. The advantages include (a) they are quick to use, in general, and (b) they do not presuppose the examination as to whether the samples agree with the assumed distributions. On the other hand these tests, in general, are (a) not so efficient and (b) biased when taking into account a wide range of alternatives. However in many cases of practical applications we have some restricted sets of types of alternatives on the one hand and we may refine the test on the other hand. These two circumstances provide a possibility of diminishing the distance between parametrical and nonparametrical procedures. This process has already been treated in the literature; the following considerations are some approaches from one side of the question.

We speak first of the possibility of refinement by the *use of a pair of statistics instead of one statistic*. The practical application of a pair of statistics seems to be easy enough, although to go over to the use of three statistics seems to be far too complicated.

1.3. In section 2 we consider the refinement property of the test based on a pair of statistics, which will be obvious qualitatively. For a more quantitative treatment, that is, for the problem of efficiency, we shall return later in some cases.

In section 3 we present some joint distributions which are related to the two-sample test of Smirnov and to the Galton test. We mention also a modification of the Smirnov statistic for the case of nearly equal sample sizes for which the distribution can be calculated easily. We then make some further remarks concerning the joint distribution and the limiting processes of the problems treated.

Finally, in section 4 we make two remarks on the two-dimensional case. In this section there is a limiting distribution theorem for the maximum of the sums of