

CURVES AS PARAMETERS, AND TOUCH ESTIMATION

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1. Introduction

The aims of this paper are two-fold: to make clear that certain problems of interpretation which arise in estimating power spectra of stationary time series are entirely analogous to problems of interpretation which arise in carrying out two very elementary and classical statistical procedures, and to develop new concepts of estimation which clarify the interpretation of all three instances. En route, we shall have to give some attention to relation of asymptotic results to practical application.

The nature of the argument with which we are concerned is such that it seems best to begin with two of the most classical situations of statistics, unrestricted regression and the construction of histograms, and to develop the necessary new concepts and attitudes as we progress step by step, rather than to introduce the new formal concepts before motivation and illustration are available.

These concepts are stated, and the nature of the results obtained outlined, in the last section.

2. Regression

A simple regression problem may be described as (i) an attempt to "predict" a y from one or more of the x as well as possible, (ii) an attempt to estimate $\text{ave } \{y|x\}$ as well as possible, or (iii) an attempt to estimate some other conditional typical value, such as the conditional median, as a function of x . The differences between these three alternatives are far less significant than the distinction between three other alternatives which cut across the first three:

- (1) the functional form to be used is simple, and is prescribed in advance,
- (2) the functional form to be used is to be simple, but is not prescribed,
- (3) the functional form is not prescribed, and need not be simple, though it is presumably continuous.

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